METHOD AND DEVICE FOR MULTIPLE INPUT/MULTIPLE OUTPUT TRANSMIT AND RECEIVE WEIGHTS FOR EQUAL-RATE DATA STREAMS

FIELD OF THE INVENTION

In general, the invention relates to the field of communication systems. More specifically, the invention relates to strategies for wireless Multiple-Input/Multiple-Output (MIMO) communications and in particular, to establishing transmit and receive weighting matrices for use at the transmitter antenna array and the receiver antenna array.

BACKGROUND OF THE INVENTION

In a wireless communication system, a major design challenge is to maximize system capacity and performance. One such communication design known in the art is Multiple-Input/Multiple Output (MIMO), and is a means of transmitting multiple data streams on the same time-frequency channel to a single receiver. The MIMO strategy involves deploying multiple antennas on both the transmitter and the receiver.

In environments having rich multipath scattering, large increases in capacity can be achieved through the use of MIMO and appropriate transmit and receive signal processing techniques. It is well known that MIMO wireless channels have significantly higher capacities than single-input single-output wireless communication channels, which has motivated the design of wireless communication systems with multiple antennas. Current system algorithms designed to achieve high capacity include spatial multiplexing, space-time coding and adaptive modulation.

These MIMO systems include the various BLAST (Bell-labs LAyered Space-Time) type techniques proposed by Lucent as a subset. The BLAST type techniques however, do not use the channel knowledge at the transmitter and thus are sub-optimal.

For MIMO systems, the maximum theoretical system capacity can be achieved using transmit and receive weights based on the Singular Value Decomposition (SVD) combined with a water-pouring (optimally assigning power to the individual data streams) strategy for determining the optimal data rates and power distribution. This strategy for maximizing capacity must adaptively control not only the number of independent data streams to be formed but also the choice of modulation and coding to be used on each stream. However, even for full-rank channels (as are known in the art), there are cases where a fixed number of data streams having the same modulation type is desirable, for example to avoid having to make a complex real-time decision on both the number of streams and the modulation/coding type on each stream. Consequently, using the SVD weights for a fixed number of equal-rate data streams is not the best option because fixing the modulation type will not take advantage of the unequal signal-to-noise ratios at the outputs of the receive array.

Therefore, it would be desirable to have a method and device for finding transmit and receive weights that are optimized for the use of equal-rate data streams. Further, it would be desirable that these weights have a lower Bit Error Rate (BER) than weights found using the SVD approach.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is an overview diagram of a preferred embodiment of a wireless (cellular) communication system in accordance with the invention;

FIG. 2 is a block diagram illustrating a preferred embodiment of a receiving device imbedded within the communication system of FIG. 1 in accordance with the invention;

FIG. 3 is a block diagram illustrating details of an antenna combiner imbedded within the receiving device of FIG. 2 in accordance with the invention;

FIG. 3A is a block diagram illustrating a preferred embodiment of a transmitting device imbedded within the communication system of **FIG. 1** in accordance with the invention;

FIG. 4 is a block diagram illustrating one embodiment of a transmit weighting unit imbedded within the transmitting device of **FIG. 3A** in accordance with the invention;

FIG. 5 is a flow chart representation of one embodiment of a method for providing Multiple Input Multiple Output (MIMO) receive and transmit weights as performed by the antenna combiner of **FIG. 3** and transmitting device of **FIG. 3A**, for finding the MMSE linear weights, in accordance with the invention;

FIG. 6 is a flow chart representation of an alternative embodiment of a method performed by the antenna combiner of **FIG. 3** and transmitting device of **FIG. 3A**, for finding the MMSE successive cancellation weights, in accordance with the invention;

FIG. 7 is a flow chart representation of an alternative embodiment of a method performed by the antenna combiner of **FIG. 3** and transmitting device of **FIG. 3A**, for finding the MMSE successive cancellation weights, in accordance with the invention;

FIG. 8 is a flow chart representation of an alternative embodiment of a method performed by the antenna combiner of **FIG. 3** and transmitting device of **FIG. 3A**, for finding the MMSE successive cancellation weights, in accordance with the invention; and

FIG. 8A is a flow chart representation of an embodiment for the method of **FIG. 8** as performed by the antenna combiner of **FIG. 3** and transmitting device of **FIG. 3A**.

DETAILED DESCRIPTION OF THE
PRESENTLY PREFERRED EMBODIMENTS

FIG. 1 illustrates a wireless communication system **100** in accordance with one embodiment of the present invention. As shown in **FIG. 1**, a base

5   station **110** provides communication service to a geographic region known as a cell **103,102**. At least one user device **120** and **130** communicate with the base station **110**. In one embodiment of the wireless communication system **100**, at least zero external interference sources **140** share the same spectrum allocated to the base station **110** and subscriber devices **120** and **130**. The

10 external interference sources **140** represent an unwanted source of emissions that interferes with the communication process between the base station **110** and the user devices **120** and **130**. The exact nature and number of the external interference sources **140** will depend on the specific embodiment of the wireless communication system **100**. In the embodiment shown in **FIG. 1**,

15 an external interference source will be another user device **140** (similar in construction and purpose to user device **120**) that is communicating with another base station **112** in the same frequency spectrum allocated to base station **110** and user devices **120** and **130**. As shown in **FIG. 1**, user devices **120** have a single transmit antenna **101**, while user devices **130** have at least

20 one antenna **101**. One embodiment of the invention provides that the user devices **120** and **130**, as well as the base station **110** may transmit, receive, or both from the at least one antenna **101**. An example of this would be a typical cellular telephone. Additionally, one embodiment of the invention can be implemented as part of a base station **110** as well as part of a user device

25 **120** or **130**. Furthermore, one embodiment provides that user devices as well as base stations may be referred to as transmitting units, receiving units, transmitters, receivers, transceivers, or any like term known in the art, and alternative transmitters and receivers known in the art may be used.

FIG. 2, is a block diagram illustrating one embodiment of a receiving

30 device **200** imbedded within the wireless communication system **100** of **FIG. 1**, in accordance with the present invention. The receiving device **200**

includes at least one antenna **101** wherein the outputs of the antennas are each provided to a receiving unit **201**. The outputs of the receiving units **201** are provided to at least one antenna combiner **202**. The signals from the receiving units **201** are also fed into a combiner controller **210**, which may

5   regulate the operation of the at least one antenna combiner **202**. The signals from the receiving units **201** may also be fed into a channel estimation device **208**. A pilot symbol generator **212** generates pilot symbol information that is used by the combiner controller **210** to control the antenna combiner **202**. The pilot symbol information generated by the pilot symbol generator **212** is

10   also used by the channel estimation device **208** to estimate a time-varying frequency response of the transmitting devices of wireless communication system. The output of the antenna combiner **202** is fed into an information decoding unit **206**, which decodes the antenna combiner output **204** and generates data information **213** that was received by the antennas **101**.

15   **FIG. 3** is a block diagram illustrating details of the antenna combiner **202** of receiving device **200** of **FIG. 2**. In one embodiment, antenna combiner **202** can be coupled to the receiving units **201**, which in turn are coupled to the antennas **101**. The receiving units **201** may include radio frequency pre-amplifiers, filters, and other devices that can be used to convert a radio

20   frequency signal received by the antenna **101**, to a digital stream of baseband equivalent complex symbols. As illustrated in **FIG. 2**, the output (y) of the ith receiving unit **201** (where i is an integer between 1 and M inclusive, and M is the total number of antenna **101** elements) may be mathematically denoted by $y_i(k)$, where k and i are integers, and is provided to the antenna combiner **202**.

25   The antenna combiner **202** can be in the form of at least one complex multipliers **302** that multiply the output of each receiving unit **201** by a weight (w) **304** mathematically denoted as $w_i(k)$. A combiner **306** may sum the outputs of the at least one complex multipliers **302**. For one embodiment of the invention, the combiner controller **210** of **FIG. 2** controls the values of the

30   weights **304**.

FIG. 3A is a block diagram illustrating one embodiment of a transmitting device **3A00** imbedded within the wireless communication system **100** of FIG. **1**, in accordance with the present invention. The transmitting device **3A00** may include at least one antenna **101** wherein the inputs to the

5   antennas may be provided from at least one transmitting unit **3A01**, and at least one transmit combiner **3A03**. The inputs to the transmit combiner **3A03** may be provided from at least one transmit weighting unit **3A05**. At least one Information Bit Stream **3A13** may be encoded by an Information Encoding Unit **3A07** and converted into a Data Stream **3A10**. The Data Stream **3A10** is

10 weighted by transmit weighting unit **3A05** and the outputs of the transmit weighting unit **3A05** may be sent to the transmit combiner **3A03**. The Transmit Weighting Unit Controller **3A11** may be provided with channel state information **3A12**, which may be used by the Transmit Weighting Unit Controller **3A10** to control the operation of transmit weighting units **3A05**.

15 FIG. **4** is a block diagram illustrating one embodiment of the transmitting weighting unit **3A05** of FIG. **3A**, in accordance with the present invention. The symbol (x) on data stream i at time k, $x_i(k)$ **410** is weighted **403** by a transmit weight **404**. The transmit weight **404** value may be dependant on which antenna the transmit weight **404** is associated with. In equation

20 form, an antenna m's signal (v) for data stream i at time k **420** may be expressed as $v_{i,m}(k) = V_{i,m}(k) x_i(k)$. The antenna m's signal at time k for data stream i may be sent to the transmit combiner **3A03** for antenna m in order to combine all the data streams before radiating the signal out of antenna m.

FIG. **5** is a flow chart representation of a method for providing multiple

25 input multiple output MIMO receive weights as can be performed by the antenna combiner of FIG. **3**, and the transmitting device **3A00**, in accordance with the invention.

One embodiment of the invention may transmit and receive N_s data streams, wherein each data stream can have the same modulation type with

30 M_T transmit antennas and M_R receive antennas. The present invention may

provide an alternative to the SVD weights that are known in the art. One
embodiment of the present invention can minimize the mean square error
between the receive array outputs and the transmitted symbols. Using the
SVD weights for a fixed number of equal-rate data streams may not be the

5   best option because fixing the modulation type will not take advantage of the
unequal signal-to-noise ratios at the outputs of the receive array. This is
because the SVD weights diagonalize the channel, which means that there is
no cross talk between the data streams. Thus MMSE weights based on the
SVD approach will not take advantage of the performance gains possible by

10 trading off the suppression of other streams with gains over noise. Further, if
the SVD weights are used while changing each streams' transmitted power so
that each data stream is received with equal energy, the optimal data stream
will be penalized while an inadequate data stream is built-up, again indicating
that this is not the best solution. One embodiment of the present invention

15 computes linear MMSE weights that are not constrained to diagonalize the
channel, thereby providing superior performance when all data streams have
the same modulation type.

        In order to compute the linear MMSE weights in accordance with one
embodiment of the present invention, a single estimate of the channel gain

20 and phase between each receive antenna and each transmit antenna may be
provided to the invention (this is also called the channel). The channel is
modeled as stationary and flat-faded. As is known in the art, such channels
occur in systems that include, but are not limited to, single carrier systems
operating in frequency non-selective channels, OFDM systems in which an

25 embodiment of the invention operates on a set of frequency-domain
subcarriers having identical frequency response characteristics, or single
carrier system in which an embodiment of the invention operates in the
frequency domain on a set of frequency-bins having identical frequency
response characteristics, or other similar systems as is known in the art. Also

provided is that both the transmitting weighting unit and receiving device may know the channel.

In one embodiment, the $M_R \times M_T$ channel matrix, $\mathbf{H}$, contains the gain and phase between each transmit and receive element. Also let the $M_T \times N_s$
5   matrix (also known as the transmit weighting matrix), $\mathbf{V}$, denote the transmit weights for the N_s streams (the uth column of $\mathbf{V}$ is denoted $\mathbf{v}_u$), and finally let the $N_s \times 1$ vector, $\mathbf{x}$, be the symbols on the N_s streams (at one particular time). Then for this embodiment, a received $M_R \times 1$ vector can be given as:

$$\mathbf{y} = \mathbf{HVx} + \mathbf{n} \qquad (1)$$

10   where $\mathbf{n}$ is a $M_R \times 1$ vector of noise with power of σ_n^2 on each of the received antennas. In one embodiment, in order to keep an average transmit power equal to one, the transmit weighting matrix may be normalized as follows:

$$\text{trace}(\mathbf{V}^H \mathbf{V}) = 1 \qquad (2)$$

where trace($\mathbf{A}$) means the sum of the diagonal elements of the square matrix
15   $\mathbf{A}$.

One embodiment of the invention finds the transmit weighting matrix and a $M_R \times N_s$ receive weighting matrix, $\mathbf{W}$, such that an estimate of the transmitted symbols, $\mathbf{r}$, is:

$$\mathbf{r} = \mathbf{W}^H \mathbf{y} \qquad (3)$$

20   (The uth column of $\mathbf{W}$ is denoted $\mathbf{w}_u$) In one embodiment of the invention, the operation of the invention may be optimized for the case where each of the N_s data streams is constrained to have the same modulation type. This embodiment of the invention minimizes the Mean Squared Error (MSE) between the estimated symbol on each stream and the transmitted symbol
25   assuming linear receive and transmit weights. In equation form, this can be expressed as:

$$\min_{\mathbf{w}_u, \mathbf{v}_u} \sum_{u=1}^{N_s} \mathbb{E}\left|r_u - x_u\right|^2 =$$

$$\min_{\mathbf{w}_u, \mathbf{v}_u} \sum_{u=1}^{N_s} \mathbb{E}\left|\mathbf{w}_u^H\left(\sum_{\ell=1}^{N_s} \mathbf{H}\mathbf{v}_\ell x_\ell + \mathbf{n}\right) - x_u\right|^2 \qquad (4)$$

where r_u is the uth element of the vector $\mathbf{r}$ and x_u is u^{th} element of the vector $\mathbf{x}$. Using $E\{x_u x_\ell^*\} = \delta(u - \ell)$, (4) becomes:

$$\min_{\mathbf{w}_u, \mathbf{v}_u} \left\{ \begin{array}{l} \sum_{u=1}^{N_s} \mathbf{w}_u^H \left(\sum_{\ell=1}^{N_s} \mathbf{H}\mathbf{v}_\ell \mathbf{v}_\ell^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R}\right)\mathbf{w}_u \\ - \sum_{u=1}^{N_s} \mathbf{w}_u^H \mathbf{H}\mathbf{v}_u - \sum_{u=1}^{N_s} \mathbf{v}_u^H \mathbf{H}^H \mathbf{w}_u + N_s \end{array} \right\} \qquad (5)$$

5   In addition to the minimization of the objective function in (5), the constraints on the transmit weight vectors (the columns of the matrix $\mathbf{V}$) given in (2) need to be met. One embodiment of the invention meets the constraints by modifying the objective function of (5) as follows:

$$\min_{\mathbf{w}_u, \mathbf{v}_u} \left\{ \begin{array}{l} \sum_{u=1}^{N_s} \mathbf{w}_u^H \left(\sum_{\ell=1}^{N_s} \mathbf{H}\mathbf{v}_\ell \mathbf{v}_\ell^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R}\right)\mathbf{w}_u \\ - \sum_{u=1}^{N_s} \mathbf{w}_u^H \mathbf{H}\mathbf{v}_u - \sum_{u=1}^{N_s} \mathbf{v}_u^H \mathbf{H}^H \mathbf{w}_u + N_s \\ + \gamma \left|\sum_{u=1}^{N_s} \mathbf{v}_u^H \mathbf{v}_u - 1\right|^2 \end{array} \right\} \qquad (6)$$

10   where γ is an arbitrary scaling placed on the constraint part of the objective function.

       A closed form solution to (6) does not exist for $N_s > 1$. However, (6) is in the form of an unconstrained optimization problem so a modified version of an iterative algorithm such as the gradient-based optimization approach is used

15   by one embodiment of the invention to solve for $\mathbf{w}_u$ and $\mathbf{v}_u$. The gradient of (6) with respect to $\mathbf{w}_u$ can be shown to be:

$$\nabla \mathbf{w}_u = \left(\mathbf{H}\mathbf{V}\mathbf{V}^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R}\right)\mathbf{w}_u - \mathbf{H}\mathbf{v}_u \qquad (7)$$

Because $\left(\mathbf{HVV}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}_{M_R}\right)$ is well conditioned, its inverse can be found.

Therefore $\mathbf{W}$ can be found at each iteration step as follows (i.e., by setting equation (7) equal to zero):

$$\mathbf{W} = \left(\mathbf{HVV}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}_{M_R}\right)^{-1}\mathbf{HV} \qquad (8)$$

5 The gradient of (6) with respect to $\mathbf{v}_u$ can be shown to be:

$$\nabla\mathbf{v}_u = \left(\mathbf{H}^H\mathbf{WW}^H\mathbf{H} + 2\gamma(\text{trace}(\mathbf{V}^H\mathbf{V}) - 1)\mathbf{I}_{M_T}\right)\mathbf{v}_u - \mathbf{H}^H\mathbf{w}_u \qquad (9)$$

The invention lets $\mathbf{G} = [\nabla\mathbf{v}_1 \quad \dots \quad \nabla\mathbf{v}_{N_s}]$, and then using equation (9), $\mathbf{G}$ can be expressed as:

$$\mathbf{G} = \left(\mathbf{H}^H\mathbf{WW}^H\mathbf{H} + 2\gamma(\text{trace}(\mathbf{V}^H\mathbf{V}) - 1)\mathbf{I}_{M_T}\right)\mathbf{V} - \mathbf{H}^H\mathbf{W} \qquad (10)$$

10   The above embodiment of the invention explicitly adds the constraint on the transmit weights in equation (2) to the minimization. Another option for ensuring the constraint is to use a Lagrangian multiplier, λ, as is known in the art. For this method, the gradient matrix in (10) is changed as follows:

$$\mathbf{G} = \left(\mathbf{H}^H\mathbf{WW}^H\mathbf{H} + \lambda\mathbf{I}_{M_T}\right)\mathbf{V} - \mathbf{H}^H\mathbf{W} \qquad (11)$$

15   One addition gradient is needed, the Lagrangian gradient which is:

$$g_\lambda = \text{trace}(\mathbf{V}^H\mathbf{V} - 1) \qquad (12)$$

The previous equations and calculations express a mathematical means for a method of providing linear MIMO transmit and receive weights.

The method of providing MIMO receive and transmit weights, shown in

20 the flow chart **500**, may begin by being provided a channel matrix **510**. The channel matrix $\mathbf{H}$, is a $M_R \times M_T$ matrix that contains the gain and phase between each transmit antenna (M_T is the total number of transmit antennas) and each receive antenna (M_R is the total number or receive antennas). In one embodiment of the invention, the received vector $\mathbf{y}$, is modeled as

25 $\mathbf{y}(k) = \mathbf{HVx}(k) + \mathbf{n}(k)$, where $\mathbf{V}$ is a $M_T \times N_s$ matrix of the transmit weight vectors, $\mathbf{x}(k)$ is a $N_s \times 1$ vector of the transmitted symbols at index k (e.g., time or frequency), and $\mathbf{n}(k)$ is a $M_R \times 1$ vector of noise at index k. The embodiment of the invention illustrated in **FIG. 5** may require the total transmit power to be

limited, therefore the transmit weight vectors are constrained to

$\text{trace}(\mathbf{V}^H\mathbf{V}) = 1$, where $\text{trace}(\mathbf{A})$ means to sum the diagonal elements of the

matrix $\mathbf{A}$, and superscript H means the conjugate transpose (also known as

the Hermitian) of the matrix.

5   Another embodiment of the invention may find a receive weighting

matrix (also referred to as the receive weight vectors) $\mathbf{W}$, that find an

estimate, $\mathbf{r}(k)$, of $\mathbf{x}(k)$ as a function of $\mathbf{r}(k) = \mathbf{W}^H\mathbf{y}(k)$. Next for the

embodiment **500**, block **520** initializes an iteration number $t=0$. In one

embodiment of the invention, transmit weight vectors can be initialized **530** as

10    $\mathbf{V}_0 = \dfrac{1}{\sqrt{N_s}}\begin{bmatrix} \mathbf{I}_{N_s} \\ \mathbf{0}_{(M_T - N_s)\times N_s} \end{bmatrix}$, where $\mathbf{I}_{Ns}$ is a $N_s \times N_s$ matrix of all zeros except for the

diagonal elements which are one, and $\mathbf{0}_{a\times b}$ is an $a\times b$ matrix of all zeros.

Utilizing the iteration number t, the receive weight vectors $\mathbf{W}_t$, are found **540**

as $\mathbf{W}_t = \left(\mathbf{H}\mathbf{V}_t\mathbf{V}_t^H\mathbf{H}^H + \sigma_n^2\mathbf{I}_{M_R}\right)^{-1}\mathbf{H}\mathbf{V}_t$, where $\mathbf{V}_t$ are the transmit weight vectors at

iteration number t, σ_n^2 is the noise power, and $\mathbf{I}_{MR}$ is a $M_R \times M_R$ matrix of all

15   zeros except for the diagonal elements which are one. Again utilizing the

iteration number t, the gradient matrix $\mathbf{G}_t$, is found **550** as

$\mathbf{G}_t = \left(\mathbf{H}^H\mathbf{W}_t\mathbf{W}_t^H\mathbf{H} + 2\gamma(\text{trace}(\mathbf{V}_t^H\mathbf{V}_t) - 1)\mathbf{I}_{M_T}\right)\mathbf{V}_t - \mathbf{H}^H\mathbf{W}_t$, where γ is the

constraint weight (in one embodiment, $\gamma=1$), and $\mathbf{I}_{MT}$ is a $M_T \times M_T$ matrix of all

zeros, except for the diagonal elements that are one. In another embodiment

20   of the invention, the gradient matrix is: $\mathbf{G}_t = \left(\mathbf{H}^H\mathbf{W}_t\mathbf{W}_t^H\mathbf{H} + \lambda_t\mathbf{I}_{M_T}\right)\mathbf{V}_t - \mathbf{H}^H\mathbf{W}_t$,

the Lagrangian multiplier at iteration number t can be given as $\lambda_t = \lambda_{t-1} + \alpha g_{\lambda,t-1}$

and the Lagrangian gradient at iteration number t is: $g_{\lambda,t} = \text{trace}(\mathbf{V}_t^H\mathbf{V}_t - 1)$.

If the embodiment of the invention utilizes an adaptive step size **560**,

the step size α, at iteration time t may be found **565** as the argument that

25   minimizes the following objective function, $C(\mathbf{V}_t, \mathbf{W}_t, \mathbf{G}_t, \gamma, \alpha)$ where

$$C(\mathbf{V}_t, \mathbf{W}_t, \mathbf{G}_t, \gamma, \alpha) = \mathbf{W}_t^H \mathbf{H}(\mathbf{V}_t - \alpha\mathbf{G}_t)(\mathbf{V}_t - \alpha\mathbf{G}_t)^H \mathbf{H}^H \mathbf{W}_t - \mathbf{W}_t^H \mathbf{H}(\mathbf{V}_t - \alpha\mathbf{G}_t)$$
$$- (\mathbf{V}_t - \alpha\mathbf{G}_t)^H \mathbf{H}^H \mathbf{W}_t + \gamma \mathrm{trace}((\mathbf{V}_t - \alpha\mathbf{G}_t)^H (\mathbf{V}_t - \alpha\mathbf{G}_t))$$

.

After a step size has been established, the transmit weight vectors can
be computed **570** at iteration t+1as a function of the transmit weight vectors at
5   iteration number t, the step size at iteration number t, and the gradient matrix
at iteration number t, using the equation $\mathbf{V}_{t+1} = \mathbf{V}_t - \alpha\mathbf{G}_t$. In another
embodiment of the invention, the Lagrangian multiplier may be updated with:
$\lambda_{t+1} = \lambda_t + \alpha g_{\lambda,t}$. After computing the transmit weight vectors **570**, the iteration
number is incremented by one **575** as t=t+1. If the iteration number has
10   reached an 'end' value **580**, where t_{end} is an integer designating the maximum
number of iterations, then the receive weight vectors are the receive weight
vectors at iteration number t, (**W**=**W**$_t$) **590** and the transmit weight vectors are
the transmit weight vectors at iteration number t, (**V**=**V**$_t$) and this embodiment
is completed. If the iteration number has not reached an 'end' value, **585**
15   decides if $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$, and if yes, the receive weight vectors are
chosen to be the receive weight vectors at iteration number t, (**W**=**W**$_t$) and the
transmit weight vectors are chosen to be the transmit weight vectors at
iteration number t, (**V**=**V**$_t$) **590** again completing this embodiment. For
decision block **585**, $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$ means the sum of the diagonal
20   elements of the square matrix $(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1})$ and where ε is a number indicating
how small the gradient matrix at iteration number t should get. In other words,
when $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$, the algorithm has almost converged because at the
optimal solution $\mathbf{G}_{t-1}$=**0**. If in decision block **585**, $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$, is false,
the process recedes to block **540** and the successive cancellation weights
25   method continues until all values are satisfied by the techniques described
within flow chart **500**.

At this point the transmit vectors are, if necessary, renormalized to satisfy the constraint in Equation (2). An alternate embodiment of the invention is a closed form solution to the linear transmit and receive weights that has an equal Mean Squared Error (MSE) on each or the received data streams. At the receiver, an estimate, of the transmitted symbols, $\mathbf{r}$, can be found by using a linear $M_R \times N_s$ weight matrix, $\mathbf{W}$, as follows:

$$\mathbf{r} = \mathbf{W}^H \mathbf{y} \qquad (13)$$

An MMSE approach can be used to find $\mathbf{W}$ and $\mathbf{V}$ as summarized next where a closed form expression for $\mathbf{W}$ and $\mathbf{V}$ are given. Using the SVD, $\mathbf{H}$ can be expressed as follows:

$$\mathbf{H} = \mathbf{U}_H \mathbf{S}_H \mathbf{Z}_H^H \qquad (14)$$

where $M_R \times M_R$ $\mathbf{U}_H$ and $M_T \times M_T$ $\mathbf{Z}_H$ are unitary matrices and $M_R \times M_T$ $\mathbf{S}_H$ is a matrix of all zeros except for the upper left $r_H \times r_H$ portion which is:

$$[\mathbf{S}_H]_{N_s} = \mathrm{diag}(s_{H,1}, \ldots, s_{H,r_H}) \qquad (15)$$

where r_H is the rank of $\mathbf{H}$ ($r_H \leq \min(M_T, M_R)$), $[\mathbf{A}]_\ell$ means the first ℓ rows and columns of A, and it is assumed that $s_{H,1} \geq s_{H,2} \geq \ldots \geq s_{H,r_H}$. The transmit weight matrix can also be expressed by a SVD as follows:

$$\mathbf{V} = \mathbf{U}_V \mathbf{S}_V \mathbf{Z}_V^H \qquad (16)$$

where $M_T \times M_T$ $\mathbf{U}_V$ is unitary, $N_s \times N_s$ $\mathbf{Z}_V$ is unitary, and $M_T \times N_s$ $\mathbf{S}_V$ may be a matrix of all zeros except for the upper $N_s \times N_s$ portion which contains the N_s non-zero singular values of the transmit weight matrix as follows:

$$[\mathbf{S}_V]_{N_s} = \mathrm{diag}(s_{V,1}, \ldots, s_{V,N_s}) \qquad (17)$$

One way to find the components of (16) may be to set $\mathbf{U}_V = \mathbf{Z}_H$ and the singular values can be found as the solution to:

$$s_{V,\ell}^2 = \frac{1}{\sqrt{\lambda}} \sigma_n s_{H,\ell}^{-1} - \sigma_n^2 s_{H,\ell}^{-2} \qquad (18)$$

where λ is chosen so that the following equation is satisfied (this forces the

transmit weights to have unit power):

$$\sum_{\ell=1}^{N_s} s_{V,\ell}^2 = 1 \qquad (19)$$

The MSE averaged across the data streams may be unchanged regardless of the choice of the right singular vectors of the transmit weight matrix. One option is to use the identity matrix (i.e., $Z_V=I$), however in general this choice gives unequal MSE on each data stream as will be seen. This means that the BER performance can be dominated by the data stream with the highest MSE. Thus if some unitary Z_V can be found that gives equal MSE on each stream, the BER performance will improve while the average MSE will remain unchanged.

Using $U_V=Z_H$, the received data vector from (1) can be expressed as:

$$\begin{aligned} y &= U_H S_H Z_H^H Z_V S_V Z_V^H x + n \\ &= U_H S_H S_V Z_V^H x + n \end{aligned} \qquad (20)$$

Let $N_s \times 1$ $Y_{N_s} = \{U_H^H y\}_{N_s}$ where $\{a\}_i$ means the first i elements of the vector a, then Y_{Ns} is:

$$\begin{aligned} Y_{N_s} &= \{U_H^H U_H S_H S_V Z_V^H x + U_H^H n\}_{N_s} \\ &= \{S_H S_V Z_V^H x\}_{N_s} + \{U_H^H n\}_{N_s} \\ &= D Z_V^H x + N_{N_s} \end{aligned} \qquad (21)$$

where D is a real $N_s \times N_s$ diagonal matrix equal to $\text{diag}(s_{H,1}s_{V,1},\ldots,s_{H,Ns}s_{V,Ns})$ and N_{Ns} has a covariance matrix equal to $\sigma_n^2 I_{N_s}$. (A unitary matrix times a vector of uncorrelated Gaussian random variables does not change its covariance matrix).

The MSE on stream ℓ can be shown to be:

$$MSE_\ell = 1 - Z_{V,\ell}^H \tilde{D} Z_{V,\ell} \qquad (22)$$

where $Z_{V,\ell}$ is the ℓth column of Z_V, and

$$\tilde{D} = D^2 (D^2 + \sigma_n^2 I)^{-1} \qquad (23)$$

Thus unless all of the diagonal elements of $\tilde{\mathbf{D}}$ are equal, using $\mathbf{Z}_V = \mathbf{I}$ gives unequal MSEs on each data stream. The MSE averaged across all streams can be shown to be:

$$\overline{MSE} = 1 - \text{trace}(\tilde{\mathbf{D}}) / N_s \qquad (24)$$

Therefore in order to make all data streams have the same MSE, $\mathbf{Z}_{V,\ell}$ must satisfy:

$$\mathbf{Z}_{V,\ell}^H \tilde{\mathbf{D}} \mathbf{Z}_{V,\ell} = 1 - \overline{MSE} = \text{trace}(\tilde{\mathbf{D}}) / N_s$$
$$\text{subject to } \mathbf{Z}_V \mathbf{Z}_V^H = \mathbf{Z}_V^H \mathbf{Z}_V = \mathbf{I}_{N_s} \qquad (25)$$

One type of matrix that solves (25) is the DFT matrix (another choice for when N_s is a power of four is the Hadamard matrix where the Hadamard matrix is known in the art). Thus the linear transmit weights that minimize the average MSE on the data streams and also have the same MSE on each stream are:

$$\mathbf{V} = \mathbf{V}_H \mathbf{S}_V \mathbf{F}_{N_s} \qquad (26)$$

where $\mathbf{S}_V$ is given by (17) and (18) and $\mathbf{F}_{N_s}$ is the $N_s \times N_s$ DFT matrix normalized such that $\mathbf{F}_{N_s}^H \mathbf{F}_{N_s} = \mathbf{I}$. With the transmit weights chosen according to (26), the N_s streams can be recovered without any inter-stream interference as:

$$\mathbf{r} = \mathbf{F}_{N_s}^H \mathbf{D}^{-1} \mathbf{Y}_{N_s} \qquad (27)$$

Another embodiment of the invention provides an alternative approach to calculating transmit and receive weights for MIMO equal rate data streamsand is illustrated as **FIG. 6**. The embodiment illustrated as flow chart **600**, finds the successive cancellation weights that minimize the MSE between the estimated symbols and the true symbols on each stream. The advantage of successive cancellation weights over linear weights is that some of the receive array's degrees are freedom are recovered by sequentially subtracting out the estimated contribution from the decoded streams. For example if there are three streams and three receive antennas, when decoding the first stream, two degrees of freedom in the array are needed to

null the other two streams. For the second stream, however, only one degree of freedom is needed to null the third stream since the decoded first stream is subtracted out. The extra degree of freedom then can be used in a max-ratio sense to provide an additional gain against noise. Finally, the third stream

5   has all of the degrees of freedom in the array available to provide a gain against noise.

In one embodiment, for the successive cancellation method, the estimated symbols are given as:

$$r_u = \mathbf{w}_u^H \left(\mathbf{y} - \sum_{\ell=1}^{u-1} \mathbf{Hv}_\ell \hat{x}_\ell \right), \text{ where } \hat{x}_\ell = slice(r_\ell) \quad (28)$$

10   where in one embodiment $slice(r_\ell)$ means to choose the closest signal constellation point to r_ℓ. In another embodiment, $slice(r_\ell)$ refers to any technique that involves decoding a plurality of received symbols r_ℓ into a bit stream, and then mapping the resulting bit stream to a signal constellation point.

15   The MMSE successive cancellation weights can be found as the solution to the following minimization problem:

$$\min_{\mathbf{w}_u, \mathbf{v}_u} \sum_{u=1}^{N_s} E|r_u - x_u|^2 =$$
$$\min_{\mathbf{w}_u, \mathbf{v}_u} \sum_{u=1}^{N_s} E\left| \mathbf{w}_u^H \left(\sum_{\ell=u}^{N_s} \mathbf{Hv}_\ell x_\ell + \mathbf{n} \right) - x_u \right|^2 \quad (29)$$

Note that (29) differs from (4) due to the inner sum being performed from u to N_s instead of 1 to N_s as is done in (4). Using $E\{x_u(n)x_\ell^*(n)\} = \delta(u-\ell)$, (29)

20   becomes:

$$\min_{\mathbf{w}_u, \mathbf{v}_u} \left\{ \begin{array}{l} \sum_{u=1}^{N_s} \mathbf{w}_u^H \left(\sum_{\ell=u}^{N_s} \mathbf{Hv}_\ell \mathbf{v}_\ell^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right) \mathbf{w}_u \\ - \sum_{u=1}^{N_s} \mathbf{w}_u^H \mathbf{Hv}_u - \sum_{u=1}^{N_s} \mathbf{v}_u^H \mathbf{H}^H \mathbf{w}_u + N_s \end{array} \right\} \quad (30)$$

In addition to the minimization of the objective function in (30), the constraints on the transmit weight vectors given in (2) need to be met. One embodiment

of the invention meets those constraints by modifying the objective function of (30) as follows:

$$\min_{\mathbf{w}_u, \mathbf{v}_u} \left\{ \begin{array}{l} \sum_{u=1}^{N_s} \mathbf{w}_u^H \left(\sum_{\ell=u}^{N_s} \mathbf{H}\mathbf{v}_\ell \mathbf{v}_\ell^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right) \mathbf{w}_u \\ - \sum_{u=1}^{N_s} \mathbf{w}_u^H \mathbf{H}\mathbf{v}_u - \sum_{u=1}^{N_s} \mathbf{v}_u^H \mathbf{H}^H \mathbf{w}_u + N_s \\ + \gamma \left| \sum_{u=1}^{N_s} \mathbf{v}_u^H \mathbf{v}_u - 1 \right|^2 \end{array} \right\} \tag{31}$$

where γ is an arbitrary scaling placed on the constraint part of the objective function.

5

A closed form solution to (31) does not exist for $N_s > 1$, therefore a modified version of an iterative algorithm, such as a gradient-based optimization approach, is used to solve for $\mathbf{w}_u$ and $\mathbf{v}_u$. The gradient of (31) with respect to $\mathbf{w}_u$ is:

10

$$\nabla \mathbf{w}_u = \left(\sum_{\ell=u}^{N_s} \mathbf{H}\mathbf{v}_\ell \mathbf{v}_\ell^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right) \mathbf{w}_u - \mathbf{H}\mathbf{v}_u \tag{32}$$

Equation (32) cannot be put into the same form as (8) because of the sum from u to N_s in the parenthesis. Setting equation (32) equal to zero, $\mathbf{w}_u$ can be found at each iteration as:

$$\mathbf{w}_u = \left(\sum_{\ell=u}^{N_s} \mathbf{H}\mathbf{v}_\ell \mathbf{v}_\ell^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right)^{-1} \mathbf{H}\mathbf{v}_u \tag{33}$$

15   The gradient of (31) with respect to $\mathbf{v}_u$ is:

$$\nabla \mathbf{v}_u = \left(\sum_{\ell=1}^{u} \mathbf{H}^H \mathbf{w}_\ell \mathbf{w}_\ell^H \mathbf{H} + 2\gamma(\text{trace}(\mathbf{V}^H \mathbf{V}) - 1)\mathbf{I}_{M_T} \right) \mathbf{v}_u - \mathbf{H}^H \mathbf{w}_u \tag{34}$$

In another embodiment, the gradient is computed using Lagrangian multipliers as follows:

$$\nabla \mathbf{v}_u = \left(\sum_{\ell=1}^{u} \mathbf{H}^H \mathbf{w}_\ell \mathbf{w}_\ell^H \mathbf{H} + \lambda \mathbf{I}_{M_T} \right) \mathbf{v}_u - \mathbf{H}^H \mathbf{w}_u \tag{35}$$

20   One addition gradient is needed, the Lagrangian gradient which is:

$$g_\lambda = \text{trace}(\mathbf{V}^H \mathbf{V} - 1) \qquad\qquad (36)$$

The Lagrangian gradient is needed in order to update the Lagrangian multiplier, λ.

  The method for calculating the successive cancellation weights is described in **FIG. 6** as a flow chart representation **600** of a preferred embodiment of an alternative method performed by the antenna combiner of **FIG. 3** and the transmit device of **FIG. 3A**, for finding the MMSE successive cancellation weights, in accordance with the invention. **FIG. 6** begins by being provided **610** a channel matrix $\mathbf{H}$ that is a $M_R \times M_T$ matrix which contains the gain and phase between each transmit antenna (M_T is the total number of transmit antennas) and each receive antenna (M_R is the total number of receive antennas). The received vector, $\mathbf{y}$, is modeled as $\mathbf{y}(k) = \mathbf{HVx}(k) + \mathbf{n}(k)$, where for one embodiment of the invention, $\mathbf{V}$ is a $M_T \times N_s$ matrix of the transmit weight vectors, $\mathbf{x}(k)$ is a $N_s \times 1$ vector of the transmitted symbols at index (e.g., time or frequency), k, and $\mathbf{n}(k)$ is a $M_R \times 1$ vector of noise at index k. Because the total transmit power has to be limited, in one embodiment of the invention, the transmit weight vectors may be constrained as $\text{trace}(\mathbf{V}^H \mathbf{V}) = 1$, where trace ($\mathbf{A}$) means to sum the diagonal elements of the matrix $\mathbf{A}$, and superscript H means the conjugate transpose (also known as the Hermitian) of the matrix.

  An additional embodiment of the invention finds receive weight vectors which are the columns of the receive weight matrix, $\mathbf{W}$, that can be used to find an estimate, $\mathbf{r}(k)$, of $\mathbf{x}(k)$ as

$$r_u(k) = \mathbf{w}_u^H \left(\mathbf{y}(k) - \sum_{\ell=1}^{u-1} \mathbf{Hv}_\ell \hat{x}_\ell(k) \right),$$ where $\hat{x}_\ell(k) = slice(r_\ell(k))$ and $r_u(k)$ is the uth

element of $\mathbf{r}(k)$ $slice(r_\ell(k))$ may mean to choose the closest signal constellation point to $r_\ell(k)$.

  In another embodiment of the invention, the method of **FIG. 6** can initialize an iteration number t, to zero **620**, and the transmit weight vectors

630 may be initialized as $\mathbf{V}_0 = \dfrac{1}{\sqrt{N_s}}\begin{bmatrix} \mathbf{I}_{N_s} \\ \mathbf{0}_{(M_T - N_s) \times N_s} \end{bmatrix}$, where $\mathbf{I}_{Ns}$ is a $N_s \times N_s$ matrix

of all zeros except for the diagonal elements which are one, and $\mathbf{0}_{a \times b}$ is an $a \times b$

matrix of all zeros. With the initialization complete, the method of **FIG. 6**

computes the receive weight vectors $\mathbf{W}_t$ at iteration number t **640**, are found,

for u=1 to N_s, as $\mathbf{w}_{t,u} = \left(\displaystyle\sum_{\ell=u}^{N_s} \mathbf{H}\mathbf{v}_{t,\ell}\mathbf{v}_{t,\ell}^H\mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right)^{-1} \mathbf{H}\mathbf{v}_{t,u}$ where $\mathbf{w}_{t,u}$ is the uth

column of $\mathbf{W}_t$, $\mathbf{v}_{t,u}$ are the transmit weight vectors at iteration number t, σ_n^2 is

the noise power, and $\mathbf{I}_{MR}$ is a $M_R \times M_R$ matrix of all zeros except for the

diagonal elements which are one.

   Flow chart **600** proceeds to compute the gradient vectors at iteration

number t **650**. The gradient vectors, $\mathbf{g}_{t,u}$, are found using the equation

$\mathbf{g}_{t,u} = \left(\displaystyle\sum_{\ell=1}^{u} \mathbf{H}^H \mathbf{w}_{t,\ell}\mathbf{w}_{t,\ell}^H\mathbf{H} + 2\gamma(\mathrm{trace}(\mathbf{V}_t^H\mathbf{V}_t) - 1)\mathbf{I}_{M_T} \right) \mathbf{v}_{t,u} - \mathbf{H}^H\mathbf{w}_{t,u}$, where γ can be

the constraint weight (in one embodiment of the invention, γ=1), and $\mathbf{I}_{MT}$ is a

$M_T \times M_T$ matrix of all zeros, except for the diagonal elements which are one. In

another embodiment of the invention, the gradient vectors may be found

according to: $\mathbf{g}_{t,u} = \left(\displaystyle\sum_{\ell=1}^{u} \mathbf{H}^H \mathbf{w}_{t,\ell}\mathbf{w}_{t,\ell}^H\mathbf{H} + \lambda_t \mathbf{I}_{M_T} \right) \mathbf{v}_{t,u} - \mathbf{H}^H\mathbf{w}_{t,u}$. It is by computing

the gradient vectors, $\mathbf{g}_{t,u}$, instead of the gradient matrix, $\mathbf{G}_t$, that distinguishes

the successive cancellation weights method of **FIG. 6** from the linear MMSE

weights method of **FIG. 5**.

   If a step size has not been calculated prior to **660**, the step size α, at

iteration time t may be found **665** as the argument that minimizes the following

objective function, $C(\mathbf{V}_t, \mathbf{W}_t, \mathbf{G}_t, \gamma, \alpha)$ where

$$
C(\mathbf{V}_t, \mathbf{W}_t, \mathbf{G}_t, \gamma, \alpha) = \left\{
\begin{array}{l}
\displaystyle\sum_{u=1}^{N_s} \mathbf{w}_{t,u}^H \left(\sum_{\ell=u}^{N_s} \mathbf{H}(\mathbf{v}_{t,u} - \alpha\mathbf{g}_{t,u})(\mathbf{v}_{t,u} - \alpha\mathbf{g}_{t,u})^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right) \mathbf{w}_{t,u} \\[4mm]
\displaystyle - \sum_{u=1}^{N_s} \mathbf{w}_{t,u}^H \mathbf{H}(\mathbf{v}_{t,u} - \alpha\mathbf{g}_{t,u}) - \sum_{u=1}^{N_s} (\mathbf{v}_{t,u} - \alpha\mathbf{g}_{t,u})^H \mathbf{H}^H \mathbf{w}_{t,u} + N_s \\[4mm]
\displaystyle + \gamma \left| \sum_{u=1}^{N_s} (\mathbf{v}_{t,u} - \alpha\mathbf{g}_{t,u})^H (\mathbf{v}_{t,u} - \alpha\mathbf{g}_{t,u}) - 1 \right|^2
\end{array}
\right\}.
$$

After a step size has been established, the transmit weight vectors are computed **670** at iteration t+1 as a function of the transmit weight vectors at iteration number t, the step size at iteration number t, and the gradient vectors at iteration number t, using the equation $\mathbf{v}_{t+1,u} = \mathbf{v}_{t,u} - \alpha\mathbf{g}_{t,u}$, where u=1 to N_s. In another embodiment of the invention, the Lagrangian multiplier is updated with: $\lambda_{t+1} = \lambda_t + \alpha g_{\lambda,t}$.

After computing the transmit weight vectors **670**, the iteration number may be incremented by one **675** as t=t+1. If the iteration number has reached an 'end' value **680**, where t_{end} is an integer designating the maximum number of iterations, then the receive weight vectors may be chosen to be the receive weight vectors at iteration number t, ($\mathbf{W}$=$\mathbf{W}_t$) and the transmit weight vectors may be chosen to be the transmit weight vectors at iteration number t, ($\mathbf{V}$=$\mathbf{V}_t$) **690** completing this embodiment. If the iteration number has not reached an 'end' value, **685** decides if $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$, and if yes, the receive weight vectors are the receive weight vectors at iteration number t, ($\mathbf{W}$=$\mathbf{W}_t$) and the transmit weight vectors are the transmit weight vectors at iteration number t, ($\mathbf{V}$=$\mathbf{V}_t$) **690** again completing this embodiment. For decision block **685**, $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$ means the sum of the diagonal elements of the square matrix $(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1})$ and where ε is a number indicating how small the gradient matrix at iteration number t should get. In other words, when $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$, the algorithm has almost converged because at the optimal solution, $\mathbf{G}_{t-1}$=**0**. If in decision block **685**, $\mathrm{trace}(\mathbf{G}_{t-1}^H \mathbf{G}_{t-1}) < \varepsilon$, is false,

the process recedes to block **640** and the successive cancellation weights method continues until all values are satisfied by the techniques described within flow chart **600**. At this point the transmit vectors may be renormalized, if necessary, to satisfy the constraint in Equation (2).

5   **FIG 7** shows a flowchart of another embodiment of the invention **700**. In this embodiment the gradient matrix or vectors may never be calculated, but the transmit and receive linear and successive cancellation weights can be found through means of iteration. Initially, **710** one embodiment of the invention may be provided a channel matrix. The iteration number, t **720**, may next be set to 0 and the transmit and receive vectors can be initialized **730**. In one embodiment of the invention, the transmit and receive vectors can be initialized to:

$$\mathbf{V}_0 = \frac{1}{\sqrt{N_s}}\begin{bmatrix} \mathbf{I}_{N_s} \\ \mathbf{0}_{(M_T - N_s) \times N_s} \end{bmatrix}$$

For the linear weights, the receive weights are initialized to:

$$\mathbf{W}_0 = \left(\mathbf{H}\mathbf{V}_0\mathbf{V}_0^H\mathbf{H}^H + \sigma_n^2\mathbf{I}_{M_R}\right)^{-1}\mathbf{H}\mathbf{V}_0$$

For the successive cancellation weights, the receive weights are initialized to:

$$\mathbf{w}_{0,u} = \left(\sum_{\ell=u}^{N_s}\mathbf{H}\mathbf{v}_{0,\ell}\mathbf{v}_{0,\ell}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}_{M_R}\right)^{-1}\mathbf{H}\mathbf{v}_{0,u}$$

Additionally the objective function at iteration 0, C_0, is set to an arbitrary large number. (In one embodiment C_0=infinity.)

20   The transmit weight vectors at iteration number $t+1$ may next be computed **740** as a function of the transmit weight vectors at time t, the receive weight vectors at iteration number t, and the channel matrix as follows (where γ is the constraint weight):

For the linear weights:

25     $$\mathbf{V}_{t+1} = \left(\mathbf{H}^H\mathbf{W}_t\mathbf{W}_t^H\mathbf{H} + 2\gamma(\text{trace}(\mathbf{V}_t^H\mathbf{V}_t) - 1)\mathbf{I}_{M_T}\right)^{-1}\mathbf{H}^H\mathbf{W}_t$$

For the successive cancellation weights:

$$\mathbf{v}_{t+1,u} = \left(\sum_{t=1}^{u} \mathbf{H}^H \mathbf{w}_{t,\ell} \mathbf{w}_{t,\ell}^H \mathbf{H} + 2\gamma (\text{trace}(\mathbf{V}_t^H \mathbf{V}_t) - 1) \mathbf{I}_{M_T} \right)^{-1} \mathbf{H}^H \mathbf{w}_{t,u}$$

Next, the receive weight vectors at iteration *t*+1 may be computed as a function of the transmit weight vectors **750** at iteration number *t*+1, the noise power, and the channel matrix as follows:

For the linear weights:

$$\mathbf{W}_t = \left(\mathbf{H} \mathbf{V}_t \mathbf{V}_t^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right)^{-1} \mathbf{H} \mathbf{V}_t$$

For the successive cancellation weights:

$$\mathbf{w}_{t,u} = \left(\sum_{\ell=u}^{N_s} \mathbf{H} \mathbf{v}_{t,\ell} \mathbf{v}_{t,\ell}^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right)^{-1} \mathbf{H} \mathbf{v}_{t,u}$$

The objective function at iteration number *t*+1 may next be computed **760** as a function of the transmit and receive weight vectors at iteration number *t*+1, the noise power, and the channel matrix as follows:

For the linear weights:

$$C_{t+1} = N_s + \mathbf{W}_{t+1}^H \mathbf{H}(\mathbf{V}_{t+1})(\mathbf{V}_{t+1})^H \mathbf{H}^H \mathbf{W}_{t+1} - \mathbf{W}_{t+1}^H \mathbf{H}(\mathbf{V}_{t+1})$$
$$- (\mathbf{V}_{t+1})^H \mathbf{H}^H \mathbf{W}_{t+1} + \gamma\,\text{trace}((\mathbf{V}_{t+1})^H (\mathbf{V}_{t+1}))$$

For the successive cancellation weights:

$$C_{t+1} = \left\{ \begin{array}{l} \displaystyle\sum_{u=1}^{N_s} \mathbf{w}_{t+1,u}^H \left(\sum_{\ell=u}^{N_s} \mathbf{H}(\mathbf{v}_{t+1,u})(\mathbf{v}_{t+1,u})^H \mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_R} \right) \mathbf{w}_{t+1,u} \\[12pt] \displaystyle - \sum_{u=1}^{N_s} \mathbf{w}_{t+1,u}^H \mathbf{H}(\mathbf{v}_{t+1,u}) - \sum_{u=1}^{N_s} (\mathbf{v}_{t+1,u})^H \mathbf{H}^H \mathbf{w}_{t+1,u} + N_s \\[12pt] \displaystyle + \gamma \left| \sum_{u=1}^{N_s} (\mathbf{v}_{t+1,u})^H (\mathbf{v}_{t+1,u}) - 1 \right|^2 \end{array} \right\}$$

Next, **770** if the objective function at iteration *t* is less than the objective function at iteration *t*+1, then the receive weight vectors may be the receive weight vectors at iteration number *t*, (**W**=**W**ₜ) and the transmit weight vectors may be the transmit weight vectors at iteration number *t*, (**V**=**V**ₜ) **790** completing this embodiment. If the objective function at iteration *t* is not less than the objective function at iteration *t*+1, then in **775** the iteration number, *t*,

can be incremented by one. If the iteration number has reached an 'end' value **780**, where t_{end} may be an integer designating the maximum number of iterations, then the receive weight vectors may be chosen to be the receive weight vectors at iteration number t, (**W**=**W**$_t$) and the transmit weight vectors

5   may be chosen to be the transmit weight vectors at iteration number t, (**V**=**V**$_t$) **790** completing this embodiment. If the end value is not reached, then the iterative procedure returns back to **740** and performs another iteration. After the iterations are completed, the transmit vectors may need to be renormalized to meet the constraint in Equation (2).

10   A further embodiment of the invention may provide for the successive cancellation weights to find weights that maximize the theoretical capacity as described next. The capacity of the channel for a given transmit weight matrix, **V**, can be shown to be:

$$C = \log_2\left[\det\left(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{V}\mathbf{V}^H\mathbf{H}^H\right)\right] \qquad (37)$$

15   Using (14) and (16), the capacity equation in (37) becomes:

$$C = \log_2\left[\det\left(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{U}_H\mathbf{S}_H\mathbf{Z}_H^H\mathbf{U}_V\mathbf{S}_V\mathbf{Z}_V^H\mathbf{Z}_V\mathbf{S}_V^T\mathbf{U}_V^H\mathbf{Z}_H\mathbf{S}_H^T\mathbf{U}_H^H\right)\right] \qquad (38)$$

Using $\mathbf{Z}_V^H\mathbf{Z}_V = \mathbf{I}$ and making $\mathbf{U}_V = \mathbf{Z}_H$, (38) becomes:

$$C = \log_2\left[\det\left(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{U}_H\mathbf{S}_H\mathbf{S}_V\mathbf{S}_V^T\mathbf{S}_H^T\mathbf{U}_H^H\right)\right] \qquad (39)$$

In other words, the capacity is independent of the right singular vectors

20   of the transmit weight matrix. However, even though the capacity is not affected by $\mathbf{Z}_V$, different $\mathbf{Z}_V$'s will greatly affect the performance of a practical receiver depending on what type of receive algorithm is employed (e.g., successive cancellation or linear weights). Therefore a search can be performed only over $\mathbf{Z}_V$ and the resulting weights will not change the

25   theoretical capacity but will improve the receiver performance given the algorithm employed.

To maximize the capacity, the r ($r{\neq}N_s$ in general) singular values for the transmit weight vectors are selected according to the water-pouring strategy as known in the art with a total transmit power of one (this enforces the constraint trace($\mathbf{V}^H\mathbf{V}$) = 1). If the water-pouring strategy says to transmit on more streams than N_s, then the singular values of the transmit weight vectors are selected according to the water-pouring strategy using only the largest N_s singular values of $\mathbf{H}$.

To summarize, the transmit weights are expressed as:

$$\mathbf{V} = \mathbf{Z}_H\mathbf{S}_V\mathbf{Z}_V^H \qquad (40)$$

Therefore the received data vector can be expressed as:

$$\mathbf{y} = \mathbf{U}_H\mathbf{S}_H\mathbf{Z}_H^H\mathbf{Z}_H\mathbf{S}_V\mathbf{Z}_V^H\mathbf{x} + \mathbf{n} = \mathbf{U}_H\mathbf{S}_H\mathbf{S}_V\mathbf{Z}_V^H\mathbf{x} + \mathbf{n} \qquad (41)$$

Let $r{\times}1$ $\mathbf{Y}_r = \{\mathbf{U}_H^H\mathbf{y}\}_r$ where $\{\mathbf{a}\}_i$ means the first i elements of the vector $\mathbf{a}$, then $\mathbf{Y}_r$ is:

$$\begin{aligned}
\mathbf{Y}_r &= \{\mathbf{U}_H^H\mathbf{U}_H\mathbf{S}_H\mathbf{S}_V\mathbf{Z}_V^H\mathbf{x} + \mathbf{U}_H^H\mathbf{n}\}_r \\
&= \{\mathbf{S}_H\mathbf{S}_V\mathbf{Z}_V^H\mathbf{x}\}_r + \{\mathbf{U}_H^H\mathbf{n}\}_r = \mathbf{D}\mathbf{Z}_V^H\mathbf{x} + \mathbf{N}_r
\end{aligned} \qquad (42)$$

where the diagonalized channel, $\mathbf{D}$, is a real $r{\times}r$ diagonal matrix equal to diag($s_{H,1}s_{V,1},\ldots,s_{H,r}s_{V,r}$) and $\mathbf{N}_r$ has a covariance matrix equal to $\sigma_n^2\mathbf{I}_r$. Note that $s_{H,r}{\neq}0$ because the water-pouring strategy would never dictate sending power on a stream with a singular value of zero.

For the Successive Cancellation MMSE weights, in order to maximize the capacity, r must equal the number of streams that the water-pouring strategy dictates and r must be less than or equal to N_s. Assuming this is true (i.e., $r{\leq}N_s$), the Successive Cancellation MMSE weights that maximize capacity are found by solving:

$$\begin{aligned}
&\min_{\mathbf{W},\mathbf{T}} \sum_{\ell=1}^{N_s} E\left|\mathbf{w}_\ell^H\left(\mathbf{Y}_r - \mathbf{D}\sum_{p=1}^{\ell-1}\mathbf{t}_p x_p\right) - x_\ell\right|^2 \\
&= \min_{\mathbf{W},\mathbf{T}} \sum_{\ell=1}^{N_s} E\left|\mathbf{w}_\ell^H\left(\mathbf{D}\sum_{p=\ell}^{N_s}\mathbf{t}_p x_p + \mathbf{N}_r\right) - x_\ell\right| \\
&\text{subject to}: \mathbf{T}\mathbf{T}^H = \mathbf{I}
\end{aligned} \qquad (43)$$

where the $r \times N_s$ right singular matrix, $\mathbf{T}$, is $\mathbf{T} = [\mathbf{t}_1, \ldots, \mathbf{t}_{N_s}]$, t_1 through t_{Ns} are the right singular vectors, $\mathbf{T} = \mathbf{Z}_V^H$, and $M_R \times N_s$ $\mathbf{W} = [\mathbf{w}_1, \ldots \mathbf{w}_{N_s}]$.

It can be shown that the receive weight vector for stream ℓ is given by:

$$\mathbf{w}_\ell = \left(\mathbf{D} \sum_{p=\ell}^{N_s} \mathbf{t}_p \mathbf{t}_p^H \mathbf{D} + \sigma_n^2 \mathbf{I}_r \right)^{-1} \mathbf{D} \mathbf{t}_\ell \qquad (44)$$

Ignoring the constraint for now, the gradient of the objective function in (44) with respect to $\mathbf{t}_\ell$ (the ℓth column of $\mathbf{T}$) can be shown to be:

$$\nabla \mathbf{t}_\ell = \mathbf{D} \sum_{p=1}^{\ell} \mathbf{w}_p \mathbf{w}_p^H \mathbf{D} \mathbf{t}_\ell - \mathbf{D} \mathbf{w}_\ell \qquad (45)$$

Using (45), $\mathbf{T}$ can be found through a gradient search where the constraint of $\mathbf{T}\mathbf{T}^H = \mathbf{I}$ is enforced at each step. This procedure is described in a flow chart representation **900** of a preferred embodiment of the invention illustrated in **FIG. 8**.

The embodiment of the invention illustrated in **FIG. 8** may be provided a channel matrix, $\mathbf{H}$ (**910**). Next the iteration number, m, can be set to 0 (**920**). The right singular matrix at iteration number 0, $\mathbf{T}_0$, may next be initialized by choosing any arbitrary $r \times N_s$ matrix such that $\mathbf{T}_0 \mathbf{T}_0^H = \mathbf{I}$ (**930**).

Next, the receive weight vectors at iteration number m, $\mathbf{w}_{m,\ell}$, can be computed as a function of the diagonalized channel, $\mathbf{D}$, and the right singular vectors at iteration number m, $\mathbf{t}_{m,\ell}$, (**940**) as $\mathbf{w}_{m,\ell} = \left(\mathbf{D} \sum_{p=\ell}^{N_s} \mathbf{t}_{m,p} \mathbf{t}_{m,p}^H \mathbf{D} + \sigma_n^2 \mathbf{I} \right)^{-1} \mathbf{D} \mathbf{t}_{m,\ell}$ for $\ell = 1, \ldots, N_s$. Then the gradient vectors at iteration number m, $\mathbf{g}_{m,\ell}$, may be computed as a function of the diagonalized channel, the receive weight vectors at iteration number m, and the right singular vectors at iteration number m (**950**) as $\mathbf{g}_{m,\ell} = \mathbf{D} \sum_{p=1}^{\ell} \mathbf{w}_{m,p} \mathbf{w}_{m,p}^H \mathbf{D} \mathbf{t}_{m,\ell} - \mathbf{D} \mathbf{w}_{m,\ell}$ for $\ell = 1, \ldots, N_s$. If a step size, α, has not been calculated prior to (**960**) then a step size can be calculated that will minimize the objective function in (43) subject to $\mathbf{T}_m \mathbf{T}_m^H = \mathbf{I}$

(965). The next step may compute the right singular vectors at iteration number $m+1$ as a function of the right singular vectors at iteration number m, the step size at iteration number m, and the gradient vectors at iteration number m (970) as $\mathbf{t}_{m,\ell} = \mathbf{t}_{m,\ell} - \alpha \mathbf{g}_{m,\ell}$ for $\ell = 1, \ldots, N_s$. The iteration number may then be incremented by one (975) as $m = m+1$. Then (977) the right singular matrix at iteration number m, $\mathbf{T}_m$, is formed by concatenating the right singular vectors at iteration number m together ($\mathbf{T}_m = [\mathbf{t}_{m,1} |, \ldots, | \mathbf{t}_{m,N_s}]$) and a Gram-Schmidt orthogonalization (as known in the art) can be performed on the rows of the right singular matrix at iteration number m (this enforces the constraint $\mathbf{T}_m \mathbf{T}_m^H = \mathbf{I}$). The flow chart representation 900 continues as an embodiment of the invention in FIG. 8A.

In FIG. 8A, if the iteration number has reached an 'end' value 980, where m_{end} may be an integer designating the maximum number of iterations, then the right singular matrix may be the right singular matrix at iteration number m, ($\mathbf{T} = \mathbf{T}_m$) and the receive weight vectors may be chosen to be the receive weight vectors at iteration number m, ($\mathbf{w}_\ell = \mathbf{w}_{m,\ell}$) 990 completing this embodiment. If the iteration number has not reached an 'end' value, block 985 can decide if trace($\mathbf{G}_m^H \mathbf{G}_m$) $< \varepsilon$ where $\mathbf{G}_m = [\mathbf{g}_{m,1} |, \ldots, | \mathbf{g}_{m,N_s}]$, and if yes, the receive weight vectors are chosen to be the receive weight vectors at iteration number m, ($\mathbf{w}_\ell = \mathbf{w}_{m,\ell}$) and the right singular matrix is chosen to be the right singular matrix at iteration number m, ($\mathbf{T} = \mathbf{T}_m$) 990 again completing this embodiment. For decision block 985, trace($\mathbf{G}_m^H \mathbf{G}_m$) means the sum of the diagonal elements of the square matrix ($\mathbf{G}_m^H \mathbf{G}_m$) and where ε is a number indicating how small the gradient matrix at iteration number m should get. In other words, when trace($\mathbf{G}_m^H \mathbf{G}_m$) $< \varepsilon$, the algorithm has almost converged because at the optimal solution, $\mathbf{G}_m = \mathbf{0}$. If in decision block 985, trace($\mathbf{G}_m^H \mathbf{G}_m$) $< \varepsilon$, is false, the process recedes to block 940 of FIG. 8, and the successive cancellation weights method continues until all values are satisfied

by the techniques described within flow chart **900**. After completion of the technique described within flow chart **900**, the transmit weight vectors may be given as $\mathbf{V} = \mathbf{Z}_H \mathbf{S}_V \mathbf{T}$.

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The present invention may be embodied in other specific forms without departing from its spirit or essential characteristics. The described embodiments are to be considered in all respects only as illustrative and not restrictive.

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